

# APPLICATION FOR PERMIT TO BUILD

Street No. 209 K. Lot 2 1/2 Block 3  
 Owner Western Hotel Co. Address Sacramento  
 Architect \_\_\_\_\_ Address \_\_\_\_\_  
 Contractor J. J. Satt Address 2118 O.  
 Kind of Building Res. Home

Permit  
**1336**  
Date  
10/14/10  
District  
140

Foundation \_\_\_\_\_

Posts	Girder		Span		Mud Sills	
	1st Floor	2nd Floor	3rd Floor	4th Floor	5th Floor	6th Floor
Jolists	Install Joists					
Max. Span	_____					
Bearing Partitions	_____					
Non Bearing Partitions	_____					
Story Height	_____					
Outside Walls	_____					

Ceiling Jolists \_\_\_\_\_ Span \_\_\_\_\_

Roof \_\_\_\_\_ Rafters \_\_\_\_\_

Water Heater \_\_\_\_\_ Chimney \_\_\_\_\_

Size of Building—Length \_\_\_\_\_ Width \_\_\_\_\_ Height \_\_\_\_\_

It is hereby agreed that this building will be constructed in conformity with the Ordinances of the City of Sacramento and the Laws of the State of California.

ESTIMATED COST, \$ 7000

J. J. Satt  
 Owner or Owner's Representative.

Plans must be submitted \_\_\_\_\_

**Abstract.** This paper discusses the use of the bootstrap to estimate the variance of the maximum likelihood estimator of the parameters of a multivariate normal distribution. The bootstrap is compared to the asymptotic variance-covariance matrix of the maximum likelihood estimator. The bootstrap is shown to be more accurate than the asymptotic variance-covariance matrix in estimating the variance of the maximum likelihood estimator. The bootstrap is also shown to be more accurate than the asymptotic variance-covariance matrix in estimating the variance of the maximum likelihood estimator of the parameters of a multivariate normal distribution.

**Keywords:** Bootstrap, Maximum likelihood estimation, Variance estimation, Multivariate normal distribution.

**1. Introduction.** The bootstrap is a resampling technique that has become a standard tool for statistical inference. It is used to estimate the variance of a statistic by resampling from the data. The bootstrap is particularly useful when the distribution of the data is unknown or when the distribution is complex. In this paper, we discuss the use of the bootstrap to estimate the variance of the maximum likelihood estimator of the parameters of a multivariate normal distribution.

**2. Maximum likelihood estimation of the parameters of a multivariate normal distribution.** Let  $\mathbf{X} = (X_1, \dots, X_p)$  be a random vector with a multivariate normal distribution with mean vector  $\boldsymbol{\mu}$  and covariance matrix  $\boldsymbol{\Sigma}$ . The maximum likelihood estimator of  $\boldsymbol{\mu}$  is given by  $\hat{\boldsymbol{\mu}} = \bar{\mathbf{X}}$ , where  $\bar{\mathbf{X}} = (\bar{X}_1, \dots, \bar{X}_p)$  and  $\bar{X}_i = \frac{1}{n} \sum_{j=1}^n X_{ij}$ . The maximum likelihood estimator of  $\boldsymbol{\Sigma}$  is given by  $\hat{\boldsymbol{\Sigma}} = \frac{1}{n} \sum_{j=1}^n (\mathbf{X}_j - \bar{\mathbf{X}})(\mathbf{X}_j - \bar{\mathbf{X}})^T$ .

**3. Variance estimation of the maximum likelihood estimator.** The variance-covariance matrix of the maximum likelihood estimator of the parameters of a multivariate normal distribution is given by  $\text{Var}(\hat{\boldsymbol{\mu}}) = \boldsymbol{\Sigma}^{-1}$ . The variance-covariance matrix of the maximum likelihood estimator of the parameters of a multivariate normal distribution is given by  $\text{Var}(\hat{\boldsymbol{\Sigma}}) = \frac{1}{n} \text{vec}(\boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma} \boldsymbol{\Sigma}^{-1})$ .